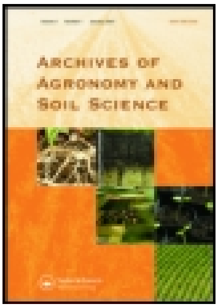


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# Evaluation of infiltration models with different numbers of fitting parameters in different soil texture classes

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In this study, the ability of eight different infiltration models (i.e. Green and Ampt, Philip, SCS (US-Soil Conservation Service), Kostiakov, Horton, Swartzendruber, Modified Kostiakov (MK) and Revised Modified Kostiakov (RMK) models) were evaluated by least-squares fitting to measured infiltration data. Six comparison criteria including coefficient of determination ($R^2$), mean root mean square error (MRMSE), root mean square error (RMSE), the $F$-statistic ($F$), $C_p$ statistic of Mallows ($C_p$) and Akaike information criterion (AIC) were used to determine the best performing model with the least number of fitting parameters. Results indicated that $R^2$ and MRMSE were not suitable for model selection. A more valid comparison was achieved by $F$, $C_p$, AIC and RMSE statistics. The RMK model including four parameters had the best performance with the majority of soils studied. RMK was better than the MK model in approximately 51.6, 57, 68.5 and 70.6% of soils, when using $F$, $C_p$, AIC and RMSE statistics, respectively, and for the other models, a higher per cent of soils was obtained. The RMK model was the best for loam, clay loam and silty clay loam soils, but the MK model was the best for silty loam soils.

**Keywords:** infiltration models; $C_p$ statistic; Akaike criterion; $F$-statistic

## Introduction

Infiltration has an important role in surface and subsurface hydrology, run-off generation, soil erosion, irrigation, etc. Hence, infiltration has been investigated by many authors throughout the last century (Kostiakov 1932; Philip 1957; Smith 1972; Mein & Larson 1973; Kao & Hunt 1996; Argyrokastritis & Kerkides 2003), and there are a large number of models for its computation. An accurate infiltration model, predicting the real infiltration correctly, is required to estimate run-off initiation time, planning of irrigation systems and management of water resources.

Throughout the last century, several infiltration models have been developed and categorized as physically based, semi-empirical and empirical models (Mishra et al. 1999). Physically based models are approximate solutions of the Richards equation (Shukla et al. 2003). Examples of these models are the models of Philip (1957), Green and Ampt (1911) and Swartzendruber (1987). Semi-empirical models employ simple forms of the continuity equation and are a compromise between empirical and physically based models. An example of semi-empirical models is the Horton (1940) model.

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Empirical models are constructed from data measured in field experiments. Parameters of empirical models have been obtained by curve fitting of the equations to measured infiltration data. Examples of such models include the SCS (USDA-NRCS 1974), Kostiakov (1932), Modified Kostiakov (MK) (Smith 1972) and Revised Modified Kostiakov (RMK) models (Parhi et al. 2007). A brief description of the infiltration models used in this study is given below.

### *Green and Ampt model (GA)*

Green and Ampt (1911) model is the earliest physically based conceptual infiltration model and expressed as:

$$I = K_s t + G \ln(1 + \frac{I}{G}) \tag{1}$$

where $I$ is the cumulative infiltration (L), $t$ is the time of infiltration ($T$), $K_s$ is a estimation of the saturated hydraulic conductivity ($LT^{-1}$) and $G$ is a parameter obtained by curve-fitting.

### *Philip model (PH)*

Philip (1957) developed an infinite-series solution to solve the non-linear partial differential Richards equation (Richards 1931), which describes transient fluid flow in a porous medium. For cumulative infiltration, the general form of the Philip model is expressed in powers of the square-root of time as:

$$I = St^{0.5} + At \tag{2}$$

where $S$ is the sorptivity ($LT^{-1/2}$) and $A$ is a parameter with dimensions of the saturated hydraulic conductivity ($LT^{-1}$). The final term in Equation (2), $At$, is not valid indefinitely, because the series diverges after a certain time.

### *Swartzendruber model (SW)*

Swartzendruber (1987) provided a series solution that holds for small, intermediate and large time:

$$I = f_c t + \frac{c}{d}[1 - \exp(-dt^{0.5})] \tag{3}$$

where $f_c$ is the final infiltration rate ($LT^{-1}$), $c$ and $d$ are empirical constants.

### *Horton model (HO)*

Horton (1940) presented a three-parameter semi-empirical infiltration model expressed as:

$$I = ct + m(1 - e^{-at}) \tag{4}$$

where the parameters $c$, $m$ ($LT^{-1}$), and $a$ ($T^{-1}$) must be evaluated using observed infiltration data.

***Kostiakov model (KO)***

A simple and general form of infiltration model presented by Kostiakov (1932) is:

$$I = \alpha_1 t^{\beta_1} \tag{5}$$

where $\alpha_1$ and $\beta_1$ are constants and evaluated using the observed infiltration data. The main limitation of using Kostiakov model is that it does describe the infiltration well at long times.

***Modified Kostiakov model (MK)***

The original Kostiakov model was modified by adding the term of ultimate infiltration capacity ($\alpha_3$) by Smith (1972), as follows:

$$I = \alpha_2 t^{\beta_2} + \alpha_3 t \tag{6}$$

where $\alpha_3$ is the final infiltration rate ($LT^{-1}$) and $\alpha_2$ and $\beta_2$ are the same as $\alpha_1$ and $\beta_1$ in the Kostiakov model.

***Revised Modified Kostiakov model (RMK)***

Recently, Parhi et al. (2007) revised the MK model and obtained a four parameter model as:

$$I = \frac{\alpha_4}{\beta_3 + 1} t^{\beta_3 + 1} + \frac{\alpha_5}{1 - \beta_4} t^{1 - \beta_4} \tag{7}$$

where $\alpha_4$, $\beta_3$, $\alpha_5$ and $\beta_4$ are parameters to be determined empirically, using measured infiltration data.

***SCS model***

The SCS model is an empirically developed approach to the water infiltration process (Jury et al. 1991), as follows:

$$I = at^b + 0.6985 \tag{8}$$

where $a$ and $b$ are constants and evaluated using the observed infiltration data.

There are several approaches for the selection of a suitable model. One of the simplest approaches is minimizing the difference between observed and predicted data to find the best model. For example, a model with higher coefficients of determination ($R^2$) may be preferred more than one with smaller $R^2$. Gifford (1976) and Machiwal et al. (2006) used the coefficient of determination ($R^2$) to compare infiltration models. Mishra et al. (2003) examined the suitability of the infiltration models with coefficient of efficiency. Turner (2006), Ghorbani Dashtaki et al. (2009) and Zolfaghari et al. (2012) used both the $R^2$ and the mean root mean square error (MRMSE) to select the best infiltration model. However, increasing the number of parameters generally improves the model performance. This occurs at the expense of a corresponding increase in the possibility of over-parameterization.

The infiltration models considered here require between two and four fitting parameters. Therefore, a better approach is to define the optimum model as the model that fits data well with the least number of fitting parameters at the same conditions. Thus, we need to use additional criteria for model comparison that have a penalty for additional fitting parameters. Several researchers have used this kind of criteria for selecting the best model. The *F*-statistic (Green & Caroll 1978) was used to compare model goodness-of-fit to soil-moisture characteristic data by Vereecken et al. (1989); Buchan et al. (1993) applied the *F*-statistic and the $C_p$ statistic of Mallows (1973) to find the best-fit particle-size distribution model. Minasny et al. (1999) and Chen et al. (1999) used the Akaike information criterion (AIC) (Carrera & Neuman 1986) to select the best predictive function of soil-moisture characteristic and the best soil hydraulic function, respectively. Hwang et al. (2002) used the *F*-, $C_p$ and AIC statistics to compare model fit to particle-size distribution data.

The objectives of this study were to test a variety of models with different underlying assumptions to determine which model represents best the soil infiltration, and to investigate if soil texture significantly affects the performance of models. To achieve these objectives, we compared the eight models described above. Six comparison techniques were considered to define the best models: the coefficient of determination ($R^2$), MRMSE, root mean square error (RMSE statistic), the *F*-statistic, $C_p$ statistic of Mallows (1973) and AIC.

## Material and methods

The infiltration data were obtained by the Double Ring method from 95 locations in 5 different provinces having a wide range of soil characteristics (Table 1): Garmaab and Khorkhore plains (Zanjan province), Ardabil plain (Ardabil province), plains of Colal and Dalki (Bushehr province), Geer and Karzin (Fars province) and Barkhaar plain (Isfahan province). The soils in these regions are classified as Mollisols, Aridisols, Inceptisols and Entisols in soil taxonomy (Soil Survey Staff 2003). The infiltration experiments were conducted until the infiltration rate reached a constant value for each soil. However, the minimum required time for each infiltration experiment was 270 min. Each infiltration measurement was replicated three times, using a double ring apparatus with outer and inner diameters of 70 and 30 cm, respectively. The physical properties of the soils are presented in Table 1. The soil texture classes of surface horizons were determined as clay loam, silty loam, loam and silty clay loam.

In this study, the RMK model with four fitting parameters was used as a reference model to compare with results from the other seven models. Five goodness-of-fit statistics listed in Table 2 ($R^2$, *F*-statistic, $C_p$ statistic, AIC and RMSE) were calculated for each

Table 1. Values of sand, silt, clay and bulk density on various soil types for the study areas.

| Province | Location | Number of tests | Sand (%) Min–Max | Silt (%) Min–Max | Clay (%) Min–Max | Bulk density ($g\ cm^{-3}$) Min–Max |
|---|---|---|---|---|---|---|
| Zanjan | Garmaab and Khorkhor | 23 | 5.8–45.4 | 32.0–61.0 | 19.2–45.2 | 1.25–1.54 |
| Fars | Gheer and Karzin | 25 | 9.8–59.3 | 28.0–63.4 | 9.0–43.7 | 1.24–1.54 |
| Bushehr | Colal plain | 8 | 10.7–33.0 | 44.1–66.7 | 16.6–34.9 | 1.41–1.58 |
| Bushehr | Dalki plain | 26 | 8.0–80.0 | 13.0–66.0 | 9.0–48.0 | 0.96–1.62 |
| Isfahan | Barkhaar plain | 7 | 32.2–67.6 | 18.0–46.4 | 14.4–28.7 | 1.35–1.54 |

Table 2. Five criteria for infiltration model comparison.

| Criteria | Equation | Explanation |
|---|---|---|
| Coefficient of determination ($R^2$) | $R^2=\left\{\frac{N(\sum I_oI_p)-(\sum I_o)(\sum I_p)}{\sqrt{[N\sum I_o^2-(\sum I_o)^2]}\sqrt{[N\sum I_p^2-(\sum I_p)^2]}}\right\}^2$ | $N$, number of observed data points. $I_o$, observed data values. $I_p$, predicted data values. j = c, comparison model. j = r, referenced model. |
| $F$-statistic (Green & Caroll 1978) | $F=[(\mathrm{SSE}_c-\mathrm{SSE}_r)/\mathrm{SSE}_r][d_r/(d_c-d_r)]$ | $P$, the number of model parameters. |
| $C_p$ statistic (Mallows 1973) | $C_p=\frac{\mathrm{SSE}_c}{\mathrm{SSE}_r/(N-P_r)}-(N-2P)$ | $P_r$, the number of parameters of the referenced model. |
| Akaike's information criterion (AIC) (Carrera & Neuman 1986) | $\mathrm{AIC}=N\{\ln(2\pi)+\ln[\mathrm{SSE}/(N-P)]+1\}+P$ | |
| RMSE[a] | $\mathrm{RMSE}=\sqrt{\frac{\sum_{i=1}^{N}(I_p-I_o)^2}{N}}$ | |
| SSE[b] | $\mathrm{SSE}_j=\sum_{i=1}^{N}(I_p-I_o)^2$ | |

Note: [a]RMSE, root mean square error; [b]SSE, sum of square error.

model and then compared. Models that best fit the data according to many goodness-of-fit statistics were considered superior to the others.

In case of the $F$-statistic, the sums of squared errors (SSE) of the reference model and the comparison model were calculated for each soil with $d_f = N - p$, where $N$ is the number of observed cumulative infiltration data points and $p$ is the number of model parameters (Table 2). A comparison model was better than the reference model when its $F$-value did not exceed the 95% significance critical $F$-value (obtainable from standard tables).

In the case of $C_p$ statistic, the $C_p$ value equals four when the RMK model is compared with itself. An alternative model was determined to fit better when the calculated $C_p$ value was more than 5% less than 4 ($C_p < 3.8$). Because $C_p$ values within 5% of four were assumed to be too similar to differentiate.

The model having the smallest AIC criterion value was selected as the best for a soil, if the AIC value of the RMK model compared with an alternative model showed a 5% decrease from its original value. The RMSE statistic is an index of the correspondence between measured and predicted data and has frequently been used as a means for evaluating the accuracy of models. A comparison model was better than the reference model, when its RMSE was smaller. In addition, the mean RMSE (MRMSE) values and mean $R^2$ values of all soils for each model were calculated. The model having the smallest MRMSE value and highest mean $R^2$ was selected as the best model according to these goodness-of-fit statistics. All models were fitted to experimental infiltration data using an iterative non-linear regression procedure, which finds the values of fitting parameters that give the best fit between model and data. The fitting process was performed using the MATLAB Software Package (The MathWorks, Inc. 2007).

## Result and discussion

### *Defining the optimum model*

Figure 1 shows an average of the cumulative infiltration in the studied sites. These curves were obtained averaging the cumulative infiltration data of the measured points for each location of the studied regions. The range of cumulative infiltration after 300 min was varied from 13 cm in Cola to 73 cm in Barkhaar sites. An example of the fitting of infiltration models to soil infiltration data is presented in Figure 2. As can be seen, infiltration can be well described with RMK and MK models.

The estimated parameter values (minimum, maximum and mean) of the infiltration models are given in Table 3.

The $R^2$ values ranged from 0.78 to 1.00 among all of the soils and models (Table 3, Figure 3). The highest $R^2$ values were associated with the RMK, MK and Swartzendruber models. The Green and Ampt model with two parameters yielded the lowest mean $R^2$ value. Among the models with three parameters, MK and Swartzendruber had mean $R^2$ higher than that of Horton model. Among the models with two parameters, mean $R^2$ was



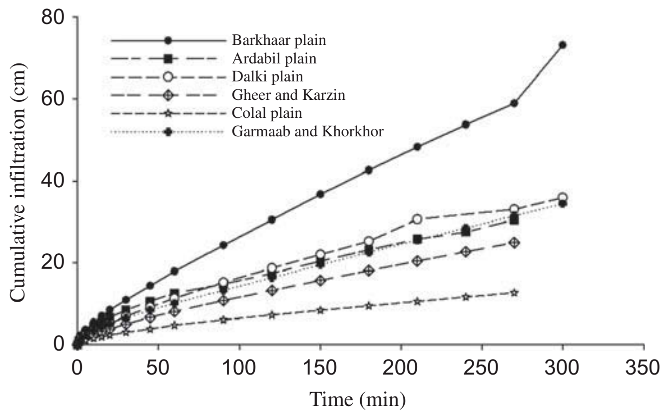


Figure 1. Average of measured cumulative infiltration in studied sites.

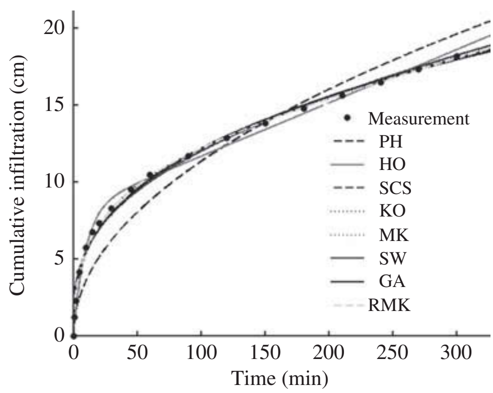


Figure 2. Comparison between the measured and fitted infiltration curve for a given soil.
Note: For abbreviations of models, refer Table 3.

Table 3. Statistics of optimized parameters of the infiltration models for all soils.

| Name | Parameters | Minimum | Maximum | Mean |
|---|---|---|---|---|
| GA | $k$ | 0.000 | 0.37 | 0.088 |
| | $G$ | 0.266 | 4509 | 141.6 |
| | $R^2$ | 0.938 | 1.00 | 0.988 |
| | RMSE | 0.001 | 3.90 | 0.340 |
| PH | $A$ | 0.000 | 1.00 | 0.083 |
| | $S$ | 0.000 | 3.13 | 0.816 |
| | $R^2$ | 0.78 | 1.00 | 0.992 |
| | RMSE | 0.000 | 4.52 | 0.384 |
| SW | $F_c$ | 0.000 | 1.00 | 0.099 |
| | $c$ | 0.140 | 3.88 | 0.920 |
| | $d$ | 0.000 | 0.68 | 0.120 |
| | $R^2$ | 0.940 | 1.00 | 0.998 |
| | RMSE | 0.004 | 4.52 | 0.321 |
| HO | $a$ | 0.000 | 1.00 | 0.110 |
| | $m$ | 0.004 | 34.8 | 2.200 |
| | $C$ | 0.001 | 58.5 | 5.250 |
| | $R^2$ | 0.960 | 1.00 | 0.996 |
| | RMSE | 0.00 | 6.16 | 0.389 |
| SCS | $a$ | 0.004 | 2.79 | 0.510 |
| | $b$ | 0.354 | 1.20 | 0.762 |
| | $R^2$ | 0.880 | 1.00 | 0.992 |
| | RMSE | 0.131 | 4.20 | 0.448 |
| KO | $\alpha_1$ | 0.288 | 1.00 | 0.762 |
| | $\beta_1$ | 0.042 | 3.11 | 0.673 |
| | $R^2$ | 0.920 | 1.00 | 0.997 |
| | RMSE | 0.000 | 4.20 | 0.427 |
| MK | $\alpha_2$ | 0.000 | 1.00 | 0.068 |
| | $\beta_2$ | 0.000 | 0.84 | 0.457 |
| | $\alpha_3$ | 0.000 | 3.23 | 0.936 |
| | $R^2$ | 0.920 | 1.00 | 0.998 |
| | RMSE | 0.000 | 4.20 | 0.276 |
| RMK | $\alpha_4$ | 0.000 | 0.81 | 0.045 |
| | $\beta_3$ | −0.997 | 3.39 | 0.435 |
| | $\alpha_5$ | 0.000 | 1.60 | 0.437 |
| | $\beta_4$ | 0.000 | 2.45 | 0.445 |
| | $R^2$ | 0.910 | 1.00 | 0.998 |
| | RMSE | 0.002 | 4.81 | 0.281 |

Note: GA = Green and Ampt; PH = Philip; KO = Kostiakov; HO = Horton; SW = Swartzendruber; MK = Modified Kostiakov; RMK = Revised Modified Kostiakov models.

higher for the Kostiakov model than that for Green and Ampt, Philip and SCS models. The ranges of $R^2$ values of each model obtained by curve fitting are presented in Figure 3. According to mean $R^2$, RMK, MK and Swartzendruber models described the data best.

According to the results obtained from mean RMSE (MRMSE) values (Table 3), the MK model provided the lowest values, indicating that infiltration was well-described by this model. Parlange and Haverkamp (1989), Ghorbani Dashtaki et al. (2009), Araghi et al. (2010) and Zolfaghari et al. (2012) also reported that the MK model was the best model for quantifying the infiltration process compared to the other infiltration models.

The relative performance of the models using the $F$-statistic, calculated with the four-parameter RMK model as the reference model, showed differences among the models

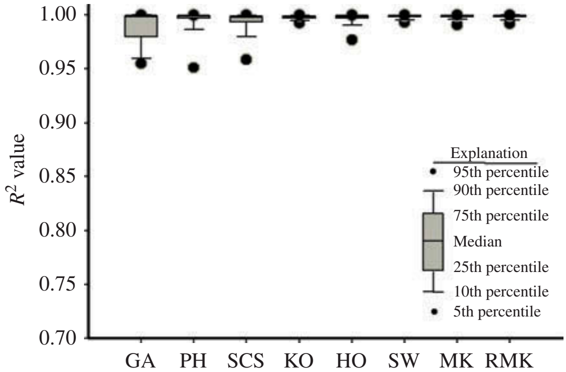


Figure 3. Box plot for $R^2$ percentiles as the goodness-of-fit of seven models for all soils.
Note: For abbreviations of models, refer Table 3.



(Figure 4). A comparison model was accepted when the $F$-value of the model was lower than the $F$-value at the 95% significance level obtained from standard tables. The MK, Kostiakov and Swartzendruber models were better than the RMK model for 48.4, 32.6 and 34.7% soils, respectively. Other models were superior compared to the RMK model for a smaller number of soils, while with the mean RMSE analysis, the MK model was better than the RMK model. The MK model was identified as the best model between two-, three- and four-parameter models according to the mean RMSE.

Results of $C_p$ statistic also showed differences between the models (Figure 4). The MK, Kostiakov and Swartzendruber models had $C_p < 3.8$ (i.e. significantly lower than the $C_p$ value of the RMK model itself) for 43.1, 30.5 and 28.4% soils, respectively, indicating that the MK, Kostiakov and Swartzendruber models were better than the RMK model for these soils. Other models were superior compared to the RMK model for smaller number of soils. On the basis of the $C_p$ statistic, the RMK model described the infiltration best for most of the soils studied (56.9% more than MK, 69.5% more than Kostiakov, 71.6% more than Swartzendruber, 82% more than Green and Ampt, 81.1% more than Horton and

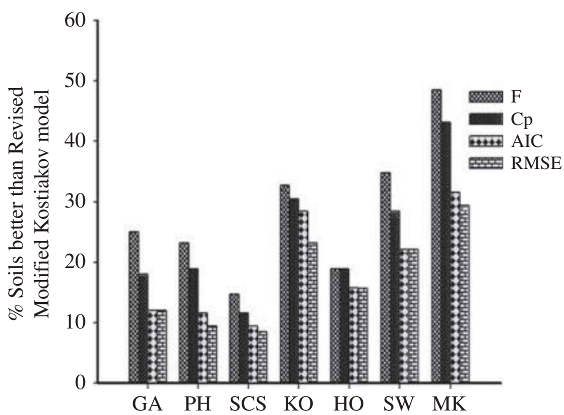


Figure 4. Statistical analyses of the infiltration models for all soils using the $F$-statistic, $C_p$ statistic, Akaike information criterion (AIC) and root mean square error (RMSE).
Note: For abbreviations of models, refer Table 3.

Philip and 88.4% more than SCS models). The SCS model with two parameters showed the lowest percentage (11.6%) of soils better than RMK model.

To confirm the performance test of *F*- and $C_p$ statistic, the AIC was used as an additional statistic. The AIC values for the MK, Swartzendruber and Kostiakov models were significantly smaller than that of the RMK model in 31.6%, 29.5% and 24.2% of soils, respectively, indicating its superiority over the RMK model for these soils (Figure 4). Other models outperformed the RMK model for a smaller number of soils.

A comparison model was accepted for the soil if its RMSE was smaller than RMSE of the reference model. Considering RMSE criterion, the RMK model was the best in 67 out of 95 soils (70.6%) compared with the MK and higher number of soils for other models. Among all the statistics, a lower number of soils better than RMK were obtained using RMSE criterion, because *F*, $C_p$ and AIC statistics have a penalty for additional fitting parameters.

The results of ranking models according to five criteria: *F*-, $C_p$ and AIC statistic, RMSE (Figure 4), MRMSE (Table 3) given in Table 4, indicated that based on *F*- and $C_p$ statistic, the goodness of cumulative infiltration can be estimated by the RMK, MK, Kostiakov, Swartzendruber, Horton, Philip, Green and Ampt and SCS models, respectively. Also, according to the AIC and RMSE statistics, the Horton model is better than the Philip model. Based on the results of model ranking given in Table 4, the SCS model obtained the lowest ranking of all models and all criteria. Also, Green and Ampt model used to predict infiltration has assumptions. The main assumptions of the Green and Ampt model are that there exists a distinct and precisely definable wetting front during infiltration (Hillel 1998).

Ghorbani Dashtaki et al. (2009) reported a better performance for Horton model than Kostiakov and Philip models. This finding is different with that obtained by Ghorbani Dashtaki et al. (2009); Parhi et al. (2007) reported a better performance for RMK model than for MK and Kostiakov models. Our results are concordant with that obtained by Parhi et al. (2007). The mean RMSE indicated a different pattern in terms of model ranking. The mean RMSE reveals that the correspondence between measured and predicted infiltration is the highest for MK model and lowest for SCS model. Results of the present study indicate that the empirical models had best fit on the double ring data, because they are on the basis of data derived from field experiments without any preassumptions. However, Swartzendruber's physically based model showed a better performance than Kostiakov, Horton and SCS models.

As result of paired *t*-tests on the $C_p$ criterion, we found that several pairs of models performed identically based on $C_p$ statistics. Philip–Swartzendruber, Philip–Horton and Swartzendruber–MK pairs performed identically based on $C_p$ statistics, which showed that pairs of models made no great differences in predicted values at measurement points

Table 4. Ranks of infiltration models using the results criteria in Table 3 and Figure 3.

| Criterion | GA | PH | SW | HO | SCS | KO | MK | RMK |
|---|---|---|---|---|---|---|---|---|
| *F*-statistic | 6 | 5 | 3 | 7 | 8 | 4 | 2 | 1 |
| $C_p$ statistic | 6 | 5 | 4 | 5 | 7 | 3 | 2 | 1 |
| AIC | 6 | 7 | 4 | 5 | 8 | 3 | 2 | 1 |
| RMSE | 6 | 7 | 4 | 5 | 8 | 3 | 2 | 1 |
| MRMSE | 4 | 5 | 3 | 6 | 8 | 7 | 1 | 2 |

Notes: RMSE, root mean square error; MRSME, mean root mean square error. For abbreviations of models, refer Table 3.

of cumulative infiltration in most of the soils. This result indicated that models within each pair (Philip–Swartzendruber, Philip–Horton and Swartzendruber–MK) may have statistically identical performance even though the equations are different.

### *Effect of soil texture on performance of models*

Figure 5 shows the $R^2$ percentiles for various soil textures. The $R^2$ percentiles were higher for the RMK, MK, Swartzendruber and Kostiakov models than other models for loam soils. The $R^2$ percentiles of RMK models were very close to 1.00 for the majority of silty clay loam soils. In addition, for silty loam and clay loam soils, RMK and MK models had a higher $R^2$.

Results of analyses on two criteria for soils with different textures are presented in Figures 6 and 7. Use of $F$-statistic indicated that MK model was better than the RMK model for 58.3% of the silty loam soils. The RMK model was superior by 69.2%, 58.9% and 51.8% for clay loam, silty clay loam and loam soils, respectively, compared with MK and other models for smaller number of soils better than the RMK model. The MK and Swartzendruber models gave equal prediction of cumulative infiltration in clay loam and silty clay loam textures. Results showed that the RMK model was the best model for most of the soils and textures, except silty loam soils.

According to $C_p$ statistic, MK model was better than RMK for 50%, 47.4%, 37.9% and 30.7% of silty loam, silty clay loam, loam and clay loam soils, respectively. While using $R^2$ analysis, the RMK and MK were the best models for silty loam and clay loam



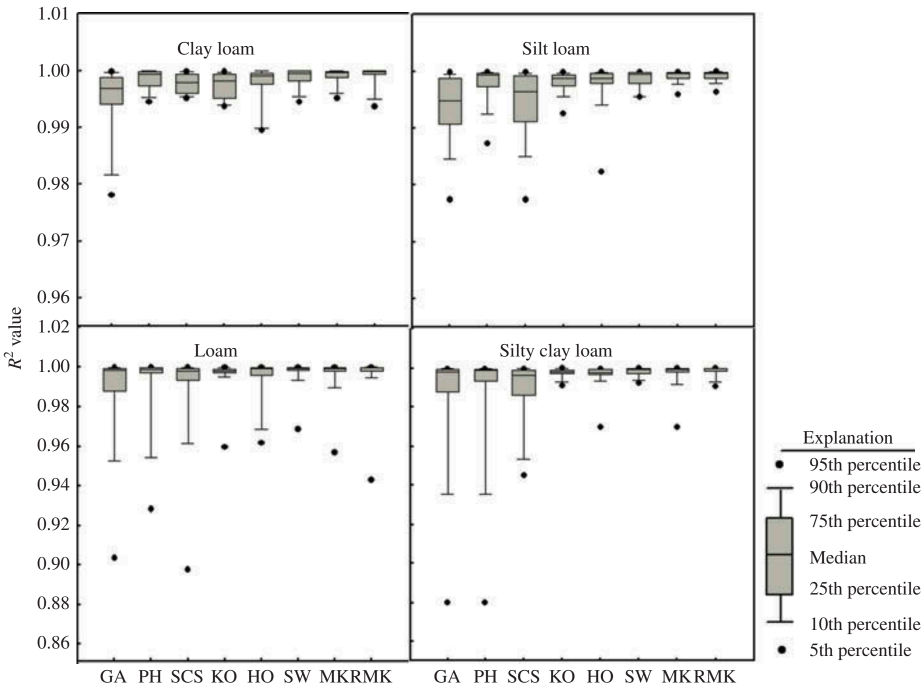


Figure 5. Box plot for $R^2$ percentiles as the goodness-of-fit of seven models for soil textural classes.
Note: For abbreviations of models, refer Table 3.

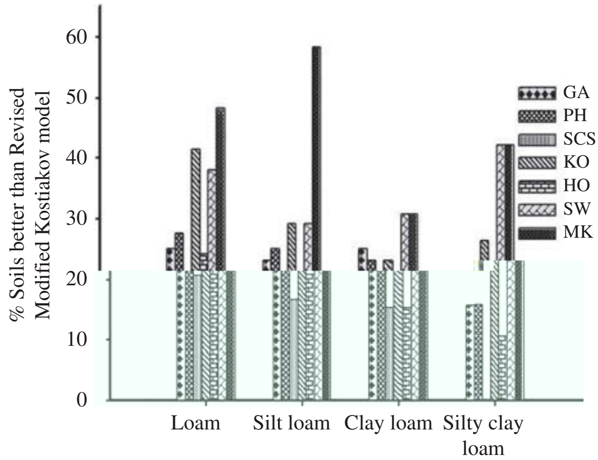


Figure 6. Statistical analyses of the infiltration models using the *F*-statistic according to soil texture.
Note: For abbreviations of models, refer Table 3.

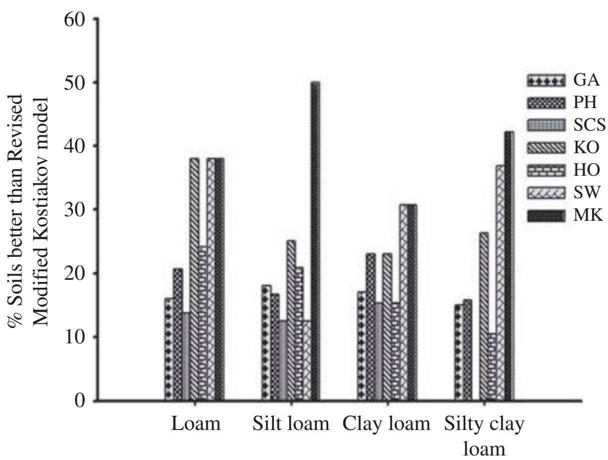


Figure 7. Statistical analyses of the infiltration models using the $C_p$ statistic (Mallows 1973), according to soil texture.
Note: For abbreviations of models, refer Table 3.

soils and the RMK, MK, Swartzendruber and Kostiakov were the best models for loam and silty clay loam soils. Other models were smaller number of soils better than RMK model.

## Conclusion

The results of this study indicated that all seven models account for >90% of the variance ($R^2$) in cumulative infiltration of majority of soils. However, $R^2$ and the mean RMSE are poor for testing the relative model fit, and should not be used for model selection. A more valid comparison is achieved by *F*-, $C_p$ and AIC statistics. The results indicated that according to *F*, $C_p$, AIC and RMSE criteria, the RMK model can be regarded as the best predictor model for most of the soils studied, while based on the mean RMSE, the MK model was the best model for prediction of cumulative infiltration.

Using paired *t*-tests for $C_p$ criteria, we found that Philip–Swartzendruber, Philip–Horton and Swartzendruber–MK model pairs can be considered to perform identically at the 95% significance level.

Texture of soil could affect the performance of cumulative infiltration models. Among four soil classes, the RMK model with four parameters showed better fits for loam, clay loam and silty clay loam soils, and worse fits than MK model for silty loam soils. In this study, among the investigated soils were not sand texture classes. Therefore, if the investigated soil textures were sand texture, the results of ranking models would be different from our results. For future studies, there would be great interest to compare performance methods for coarse-grained soils.



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